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Strategic use of payoff information in k-hop evolutionary Best-shot networked public goods game

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A R T I C L E I N F O

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A B S T R A C T

Globalization has led to increasingly interconnected interactions among individuals. Their payoffs are affected by the investment decision of themselves and their neighbors, which will cause conflicting interests between individual and social investment. Such problems can be modeled as a networked public goods game (NPGG). In this paper, we study the Best-shot NPGG model by introducing three mechanisms: k-hop, payoff information use strategy, and access cost. We use evolutionary game theory and present the k-hop evolutionary Best-shot networked public goods game (k-EBNPG) to explore the impact of these three mechanisms on social welfare. The results show that social welfare will increase with a diminishing margin as k increases while introducing the payoff information use strategy can significantly improve social welfare when $k > 1$. Finally, we study the impact of access cost on social welfare and surprisingly find that social welfare will achieve the highest when the access cost is half the investment cost.

1. Introduction

Globalization has led to increasingly interconnected interactions among nations, companies, and individuals. While such interactions have brought about convenience, they have also given rise to various security risks. Of particular concern are the threats posed by viruses in both the cyber and biological domains, which have grown in severity and scale, as exemplified by events such as the BlackByte ransomware incident [1] and the global outbreak of COVID-19 [2]. In response, companies have resorted to investing in measures such as installing firewalls to safeguard against cyber viruses, and individuals have invested in getting vaccinated to protect against biological viruses. However, because society can achieve herd immunity when the investment reaches certain constraints, and uninvested individuals can benefit directly, self-interested players have the incentive to benefit from the investment of others. This self-interested behavior, however, can lead to the failure of social investments, resulting in significant losses for the whole

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society. These problems with conflicting interests between individual and social investment can be modeled as a networked public goods game (NPGG) [3]. By studying this model, we can gain deeper insights into the essence of these issues and guide enterprises and society in devising more effective response strategies.

In a canonical version of NPGG, community members choose the amount to invest in a public good and share the subsequent value of the total efforts equally [4,5]. Many versions of NPGG have been studied, including an important variation named Best-shot NPGG, combining the Best-shot model [6], in which an individual's payoff is determined by the highest contribution rather than total efforts. In Best-shot NPGG scenarios, a high social welfare, i.e., the sum of the payoffs of all players, is more difficult to achieve [7]. As a result, the research on the performance of pure-strategy Nash Equilibrium (PNE), as measured by social welfare, has garnered significant attention from researchers. Measured by the utility of social welfare, Bramoulle et al. [8] first prove the existence of specialized equilibrium and correspond it to the maximal independent set, guiding the subsequent research on PNE. Komarovsky et al. [9] then prove the PNE of the Best-shot NPGG model is Pareto efficient and present side payments to find the optimal outcome with maximum social welfare by utilizing the potential function. And Levit et al. [10] first analyzed the balance between the effective PNE and the stable PNE in the homogeneous scenario, extending to a heterogeneous scenario by Yu et al. [11]. In addition, regarding how to improve the solution quality of PNE, i.e., the PNE with a higher social welfare, researchers introduced several mechanisms, such as the insurance [12], resource pool [13], the matching funds [14]. Different from these mechanisms on the network, Shin et al. [15] generalized the model from the direct neighbors to k-hop neighbors to increase the complexity of the neighbor structure of individuals. Apart from the utility of social welfare, the equality of social welfare is also presented to measure the PNE [16,17].

As mentioned above, there are numerous quantitative analyses of the optimal social welfare of NPGG. However, there are two limitations: 1) The Nash Equilibrium is a static result in traditional game theory. However, how the game evolves dynamically is a challenging and interesting problem, which can help researchers understand the internal mechanism of the game; 2) Unlike the utterly rational assumption in traditional game theory, in real scenarios, however, players usually have bounded rationality with limited perception and computational power. As a result, they cannot always keep the optimal decision. To address these issues on the Best-shot NPGG model, we employ the evolutionary game theory, which is widely applied to analyze the impact of mechanisms on social welfare [18–20] from the micro perspective. By investigating the Best-shot NPGG model from a microscopic perspective, we can effectively elucidate the trends in investment decision-making among individuals, providing valuable insights that can inform the investment decisions of companies and governments. Additionally, evolutionary games theory has rarely been applied in the Best-shot NPGG model, although it has been extensively used in the domains of the canonical NPGG model [21–23].

Normally, in the evolutionary game theory, there are two most commonly used mechanisms for players to update their strategies for achieving higher payoffs: (1) players who have payoff information imitate the strategy of the neighbor with the highest payoff [24], (2) players don't have payoff information so that imitate the strategy of a random neighbor with probability [25]. This selection process is crucial in determining the player's learning outcome, as it influences the information the player acquires about the game. By choosing a neighbor randomly, the player may obtain a diverse range of information useful for their learning process. Alternatively, selecting the neighbor with the highest payoff can provide more specific and focused information that may be more relevant to the player's goals. However, the use of information does not guarantee success [26,27]. Therefore, we treat the use of payoff information as an essential part of players' strategies, which can be decided and changed in evolution. Players would imitate the strategy of the neighbor with the highest payoff if they use the payoff information and emulate the strategy of a random neighbor otherwise. By introducing the payoff information use strategy, we can better understand how social learning processes unfold in complex environments. Such insights may be valuable in designing effective information dissemination and learning strategies in real-world social networks. In this paper, for a more comprehensive study, we have bifurcated the updating mechanism into two consecutive stages: learning and accessing. The present division is premised upon the assumption that players may change their preferences for using information during the decision process. Specifically, some individuals may opt to incorporate available information to inform their decision-making when selecting the learning target and not use it when choosing the accessing target. By analyzing these stages separately, we seek to provide a more comprehensive understanding of the underlying mechanisms contributing to social systems' evolution. Our approach considers the potential variability in individual preferences and behaviors, allowing us to model the complex dynamics in social evolution with greater accuracy.

In summary, our contributions are as follows:

(1) we present the k-hop evolutionary Best-shot networked public goods game (k-EBNPG) model by introducing access range (k-hop), access cost, and payoff information use strategy and analyze the impact of these introduced mechanisms on social welfare. The player's strategy is expanded with three components: investment selection, access target, and payoff information use strategy.
(2) we delineate the evolutionary process into learning and accessing stages to provide a comprehensive understanding of the complex nature of the evolutionary process, highlighting the crucial role of information utilization in shaping individual behavior and decision-making.
(3) we provide extensive numerical and simulation experiments in the lattice network to analyze the impact of k-hop, payoff information use strategy, and access costs on social welfare from a microscopic perspective. Notably, we assign the investment age to help analyze from the experimental perspective.

The rest of the paper is organized as follows. Section 2 describes our k-EBNPG model in detail. Section 3 analyzes the experimental results on the lattice network. Finally, Section 4 summarizes the research results and proposes future research directions.

2. Model

This section will describe and explain the entire k-EBNPG model from three perspectives: strategy space, payoff function, and updating mechanism.

Strategy space. To begin with, we consider the game played by n players on an undirected network $G = (V, E)$. Here, the node set V denotes the players in the game, and the edge set E indicates direct contact between players. Each player can decide whether to invest, select a player within a k-hop range to access and decide whether to use payoff information. In our model, we present the strategy $x_i = \langle a_i, h_i, F_i \rangle$ of the player $i \in V$. The following three components characterize this strategy: the investment decision of player i denoted by a_i, the player h_i selected by player i to access, and the decision of player i to use or ignore the available payoff information denoted by F_i. Following are the details:

(1) The first component is the player's investment decision $a_i \in \{0,1\}$. $a_i = 0$ represents that the investment decision is free-ride, which means the player doesn't invest and tries to benefit from following others' investments by paying the access cost. And $a_i = 1$ denotes that the investment decision of player i is to invest, paying the investment cost of the public goods and can be followed by k-hop neighbors.
(2) The second component, tightly combined with the investment decision, is the target player $h_i \in V$ selected to follow. When the player i chooses to invest, she can directly benefit from public goods without following other players. Hence we set $h_i = i$. Otherwise, if the player i chooses to free-ride to benefit from the public goods, another player $h_i \in N_k(i)/\{i\}$ within k-hop is chosen to access.
(3) The last component is the payoff information use strategy F_i, affecting the selection of target players to learn and access. We use $F_i = \left(f_{1i}, f_{2i}\right) \in \{0,1\}^2$ to denote the payoff information use strategy in the learning and accessing stages. Where $f_{1i} = 0$ (resp. $f_{2i} = 0$) indicates that the player i selects a k-hop neighbor randomly without using payoff information to learn and imitate (resp. to follow). And $f_{1i} = 1$ (resp. $f_{2i} = 1$) represents that the player i selects a k-hop neighbor with the highest payoff to learn and imitate (resp. to follow). We will elaborate on the specific details of the updating mechanism.

Payoff function. As a fundamental assumption of this paper, the payoffs of all players are considered public information. Consequently, the payoff received by a player i is solely determined by its own investment decision and the investment decision of the host player it chooses to access. Thus, we define the payoff function as follows:

(1) When a player i invests in the public good, i.e., $x_i = \langle 1, i, F_i \rangle$, she becomes an invested (IN) player by paying an investment cost c. Therefore, the payoff of an IN player i comprises the benefit of enjoying the public good, denoted by b, and the accessed income from all players who choose to access her. Formally, the payoff of an IN player i can be expressed as follows:

$$u_i\left(x_i\right) = b + m_i \cdot r - c \tag{1}$$

where $m_i = |\left\{ j \in N_k(i) \mid x_j = \langle 0, i, F_i \rangle \right\}|$ denotes the number of players who choose to access the player i. Consider that in the actual scenario, such as using public bikes [28] or renting books [29], some payment is required, denoted as r in this paper.
(2) When the player i chooses to free-ride and the player she follows, h_i, chooses to invest (i.e., $x_i = \langle 0, h_i, F_i \rangle$, $a_{h_i} = 1$), player i is considered a successful-access (SA) player. As a result, she will reap the benefits of public goods while incurring access costs r. Consequently, the payoff of a SA player i can be expressed as follows:

$$u_i\left(x_i\right) = b - r \tag{2}$$

(3) When a player i and the player h_i she follows both choose to free-ride (i.e., $x_i = \langle 0, h_i, F_i \rangle$, $a_{h_i} = 0$), the player i is a failed-access (FA) player who gains no benefits. Therefore, the payoff of a FA player i can be defined as follows:

$$u_i\left(x_i\right) = 0 \tag{3}$$

Overall, the payoff function of the player i is as follows:

$$u_i\left(x_i = \langle a_i, h_i, F_i \rangle\right) = \begin{cases} b + m_i \cdot r - c, & if\ a_i = 1 \\ b - r, & if\ a_i = 0\ and\ a_{h_i} = 1 \\ 0, & otherwise \end{cases} \tag{4}$$

Updating mechanism. Players will learn the strategies of their neighbors with probabilities. Specially, we divide the updating mechanism into learning and accessing stages for more comprehensive analysis, considering potential variations in behavior and preferences among free-riders in the context of information utilization and target selection. A randomly selected player i should choose a learning target j and decide whether to imitate the strategy of j in the learning stage. Subsequently, the player i should ensure her accessing target in the accessing stage. Details are as follows:

First, in the learning stage, a randomly selected player i must choose her learning target j in k-hop neighbors. By choosing a neighbor randomly, the player may obtain a diverse range of information useful for their learning process. Alternatively, selecting the neighbor with the highest payoff can provide more specific information that may be more relevant to the player's goals. Self-interested players will choose the learning target due to their payoff information use strategy f_{1i}. Specifically, if $f_{1i} = 0$, player i

may think a diverse range of information can bring more chances and randomly selects a k-hop neighbor j without using any payoff information. On the other hand, if $f_{1i} = 1$, player i may think more focused information can bring a higher payoff and uses the payoff information, i.e., selecting the k-hop neighbor j with the highest payoff.

Then the player i imitates the strategy of her selected learning target player j, with the probability obtained by the Fermi rule as follows:

$$P\left(x_i(t) \leftarrow x_j(t)\right) = \frac{1}{1 + exp[\frac{(u_i(t)-u_j(t))}{\varepsilon}]} \quad (5)$$

where $x_i(t)$ and $u_i(t)$ indicate the strategy and payoff of the player i at time t. $\varepsilon \geq 0$ denotes the degree of individual irrationality, and the higher ε, the higher probability for a player to imitate a lower payoff strategy. Specifically, when $\varepsilon \to 0$, players are fully rational, i.e., the learning probability is extremely high (resp. low) when the target players have higher (resp. lower) payoffs than themselves. Without losing generality, we take $\varepsilon = 0.1$ [30].

In summary, this selection process is crucial in determining the player's learning outcome, as it influences the information the player acquires about the game. The pseudo-code of the learning stage is shown in Algorithm 1.

Algorithm 1 Learning.

Require: graph G, player i, payoff information use strategy f_{1i};
Ensure: the strategy x_i and the learning target j of the input player i;
if $f = 0$ **then** #randomly choose target player without using payoff information
  randomly selects a player $j \in N_k(i)$;
else #choose the player in k-hop with the highest payoff
  selects the player $j \in N_k(i)$ and $u_j = \max_{y \in N_k(i)} u_y$;
end if
calculates the imitation probability P by the Eq. (5);
generalize a random number $p \in [0, 1]$;
if $p < P$ **then**
  the player i imitates the strategy of the selected player j, i.e., $x_i \leftarrow x_j$;
end if
Return the strategy x_i and the learning target j;

Then the player i steps into the accessing stage. In this stage, player i must select an accessing target h_i from the k-hop neighborhood based on the learned investment decision and the payoff information use strategy f_{2i}. This selection criterion is based on the assumption that the player with the highest payoff is also the most likely to bring the highest payoff in the k-hop neighborhood. So self-interested players will select the accessing target by order of the known investment player to the highest payoff player.

Specifically, if the learned investment decision of player i is to invest, i.e., $a_i = 1$, the accessing target will be player i herself, i.e., $h_i = i$. However, if the learned investment decision is to free-ride, i.e., $a_i = 0$, then player i should choose another player in the k-hop neighborhood to access. If the learned accessing target $h_i = h_j$ is within the k-hop neighborhood of player i, i.e., $h_j \in N_k(i)$, player i will not change the accessing target. This is because the accessing target is a good choice that can bring a higher payoff. On the other hand, if the learned accessing target, $h_i = h_j$, is beyond the k-hop neighborhood of player i, i.e., $h_j \notin N_k(i)$, then player i will randomly select a player q from her k-hop neighborhood, i.e., $h_i = q$, where $q \in N_k(i)$. However, if the payoff information use strategy f_{2i} of player i equals 1, she will choose the player q with the highest payoff from her k-hop neighborhood. That is, $h_i = q$, where $q \in N_k(i)$ and $u_q = \max_{y \in N_k(i)} u_y$.

In summary, the accessing stage aims to finish selecting an accessing target h_i by player i, based on her investment decision, the k-hop neighborhood, and the payoff information use strategy f_{2i}. The pseudo-code of the accessing stage is shown in Algorithm 2.

Algorithm 2 Accessing.

Require: graph G, player i, the learning target j of player i, payoff information use strategy f_{2i};
Ensure: the accessing target h_i of the input player i;
if $a_i = 1$ **then**
  Return the accessing target $h_i \leftarrow i$;
end if
if $h_j \in N_k(i)$ **then**
  Return the accessing target $h_i \leftarrow h_j$;
end if
if $f_{2i} = 0$ **then** #choose target player without using payoff information
  randomly select a player q satisfying $q \in N_k(i)$ and $q \neq j$;
else #choose the player in k-hop with the highest payoff
  select the player q satisfying $q \neq j, q \in N_k(i), u_q = \max_{y \in N_k(i)} u_y$;
end if
Return the accessing target $h_i \leftarrow q$;

The pseudo-code of the entire Monte Carlo process of the k-EBNPG model is shown by Algorithm 3: Monte Carlo simulation.

Algorithm 3 Monte Carlo simulation.

1: Build a lattice network where each node has 4 neighbors
2: **for** each player $i \in V$ **do** #Initialize
3: randomly initialize $a_i \in \{0,1\}$; $F_i \in \{0,1\}^2$; $h_i = i$ if $a_i = 1$ and $h_i = j, j \in N_k(i)$ otherwise;
4: **end for**
5: **for** each player $i \in V$ **do**
6: calculate the payoff of the player i based on the Eq. (4);
7: **end for**
8: **for** $t \in (1, T)$ **do** #Monte Carlo Simulation
9: **for** $n \in (0, |V|)$ **do**
10: randomly chooses a player i
11: the strategy and learning target $x_i, j \leftarrow Learning(G, i, f_{1i})$ #Learning stage
12: the accessing target $h_i \leftarrow Accessing(G, i, j, f_{2i})$ #Accessing stage
13: **end for**
14: **for** each player $i \in V$ **do**
15: calculate the payoff of the player i based on the Eq. (4);
16: **end for**
17: **end for**

In the random initialization phase, players randomly determine their strategies, and their subsequent payoffs are calculated after initialization, i.e., all players finish strategy determination (line 1 to line 7). In each MC step, the random selection operation would be repeated $|V|$ times to ensure that each node is selected once on average. In each selection operation, a player i would be randomly selected. Then she will go through the learning stage (line 11) and the accessing stage (line 12). At the end of each MC step, the payoff of all players should be updated according to their determined strategies (line 14 to line 16).

3. Results

In this paper, we investigate the impact of k-hop, payoff information use strategy, and access costs on social welfare in a Lattice network [4], where the population size is $n = 50 \times 50$, and the degree of nodes is $d = 4$. Specifically, without loss of generality, we set the benefit of public goods as $b = 1$ and the investment cost as $c = 0.5$ [17]. Because everybody will undoubtedly invest when the investment cost is lower than the access cost, we set the access cost $r \in [0, c]$. We conduct simulations using $T = 1000$ Monte Carlo steps to ensure network convergence to stability. And we measure the performance of evolutionary results using the average social welfare $\overline{U} = \frac{\sum_{i \in V} u_i}{|V|}$, which is an essential measurement in public goods game model [31,32] and evolutionary game model [33,34].

To study the effect of the payoff information use strategy, we conduct experiments on the lattice network with payoff information use strategies at different k and r, where all players can decide whether to use payoff information. In detail, if players decide to use payoff information, they will choose the one with the highest payoff to learn or access. Otherwise, they will select their targets of learning or accessing randomly. To study the effects of information use strategy, we compare the network with two lattice networks without such strategies, i.e., completely do not use and entirely use payoff information. In a network where players completely do not use payoff information, all players will randomly choose their targets of learning and accessing. And in a network where players entirely use payoff information, all players will choose their targets of learning and accessing with the highest payoff.

As shown in Fig. 1, it is clear that when the access cost equals the investment cost, i.e., $r = c = 0.5$, all players will invest in preventing failed access, and the average social welfare will stay at 0.5. So we only consider scenarios where the access cost is lower than the investment cost, i.e., $r < c$, in the subsequent analysis. Specifically, we find the following three phenomena from the perspectives of k-hop, payoff information use strategy, and access cost:

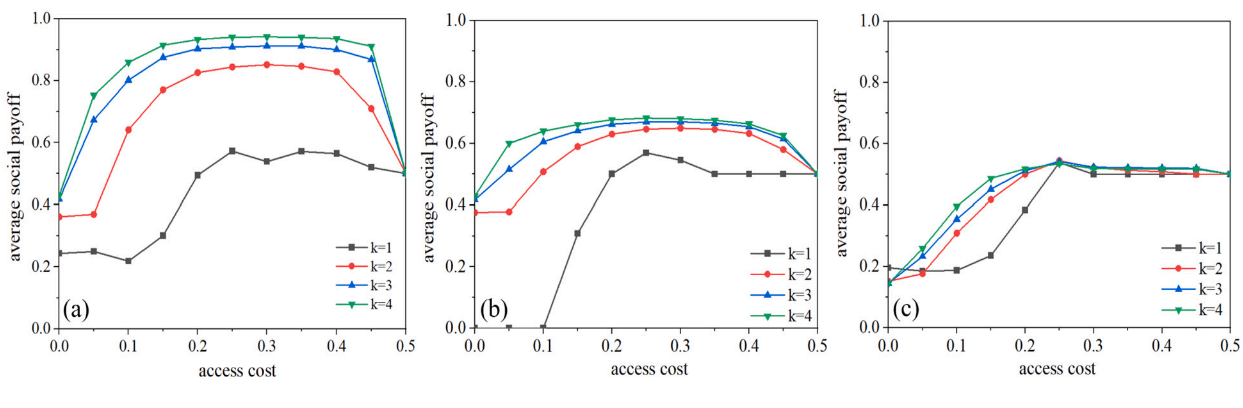

Fig. 1. Evolutionary results of average payoff with access costs on lattice network of degree 4, (a) with payoff information use strategies, (b) without using information, (c) entirely using information, and the curves in the figure indicate the k increases from 1 to 4.

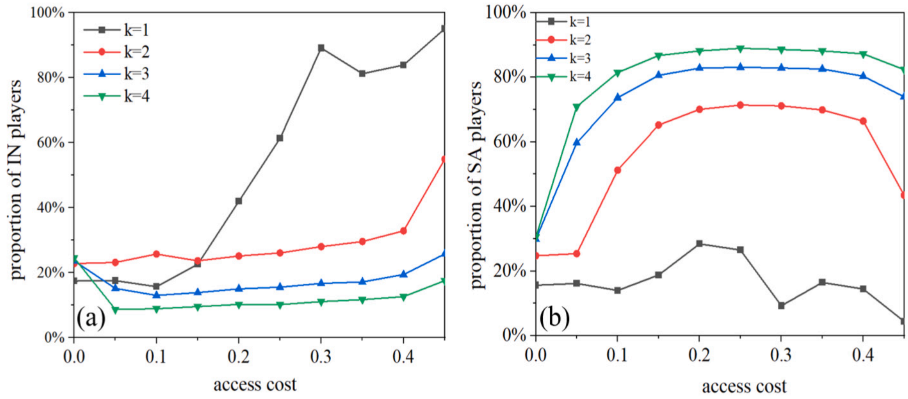

Fig. 2. Proportion of (a) IN and (b) SA players to all players on the Lattice network of degree 4 with payoff information use strategies and different k-hop. The curves in the figure indicate the k raises from 1 to 4.

(1) the impact of k-hop on social welfare. As k increases, the average social welfare experiences an upward trend, while the marginal payoff [35] decreases. The marginal payoff denotes the average expected change in a player's payoff resulting from a unit increase in k. Interestingly, the impact of payoff information use strategy and access cost on social welfare is substantially different at $k = 1$ compared to $k > 1$.
(2) the impact of payoff information use strategy on social welfare. Our results show that introducing the payoff information use strategy can significantly improve social welfare, especially when $k > 1$, compared to the scenarios where players completely do not use or entirely use payoff information.
(3) the impact of access costs on social welfare. The player's access intention generally decreases as the access cost increases. However, social welfare exhibits a trend of rising first and then falling instead of changing linearly, which is interesting.

3.1. The impact of k-hop on social welfare

In this subsection, we analyze the phenomenon (1) in two parts: the trend of social welfare and the particular performance at $k = 1$.

To begin, we define the formulation of the average social welfare to analyze its trend. Suppose there are n_{IN} IN players, n_{SA} SA players, and n_{FA} FA players. Their payoffs can be formulated as $u_{IN} = n_{IN} \cdot (b - c) + r \cdot n_{SA}$, $u_{SA} = n_{SA} \cdot (b - r)$ and $u_{FA} = 0$. Here, b, c, and r represent the benefit of enjoying the public good, investment costs, and access costs, respectively. The average social welfare $\overline{U}$, denoting the ratio of the social welfare U to the number of players n, is given by:

$$\overline{U} = \frac{U}{n} = \frac{u_{IN} + u_{SA} + u_{FA}}{n} = \frac{n_{IN}}{2n} + \frac{n_{SA}}{n} \tag{6}$$

It is found that the average social welfare is mainly affected by the ratio of IN and SA players to all players, while the percentage of SA players has a more significant impact. Therefore, to analyze the effect of k-hop on social welfare, we count the ratio of IN players and SA players to all players, i.e., $\frac{n_{IN}}{n}$ and $\frac{n_{SA}}{n}$, in the Lattice network. As shown in Fig. 2, as the value of k increases, the percentage of SA players grows due to the increasing number of available players. The improved degree of percentage of SA players is nearly equal to the decreased degree of the rate of IN players. Hence, the average social welfare increases as k increases with diminishing marginal payoff, as presented in the phenomenon (1).

Then we study the particular performance at $k = 1$. We count the common accessible targets between a player i and another player j within i's k-hop. As illustrated in Fig. 3, players can access more target players as k increases. In particular, we observe no common accessible targets between player i and player $j \in N_k(i)$ at $k = 1$, while at $k > 1$, there are at least three common accessible targets.

To further verify the analysis above, we focus on followed-access free-riders, denoting the players who successfully followed access the accessing targets of their learning targets. Specifically, if a player i is identified as a followed-access free-rider, it implies that (1) both i and her learning target, j, are free-riders, (2) j's access target h_j falls within i's access range, (3) i follows j to access h_j, i.e., $h_i = h_j$. To quantify the effect of followed-access free-riders, we calculate the proportion of these players among all players, namely the "following effect". As shown in Fig. 4, it's verified that players cannot follow their learning targets to access a known IN player when $k = 1$, and the following effect increases as k grows. This finding illustrates that when $k = 1$, a player cannot successfully follow the learning target to access the known IN player. As a result, all players rely on their payoff information use strategy to select the accessing target in the accessing stage. Thus, the average social welfare performs differently from $k > 1$, as presented in the second part of the phenomenon (1).

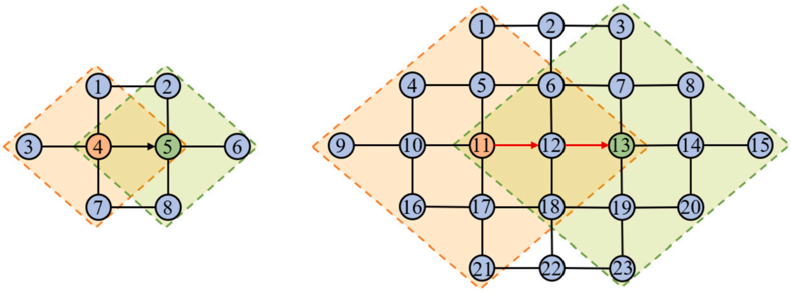

Fig. 3. Commonly accessible targets of a player i and another player j in i's k-hop on the Lattice network of degree 4 with $k = 1$ (left) and $k = 2$ (right).

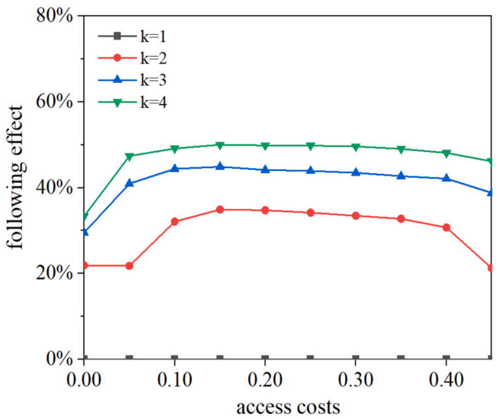

Fig. 4. The following effect of players on the Lattice network of degree 4 with payoff information use strategy, with the curves in the figure indicating k from 1 to 4.

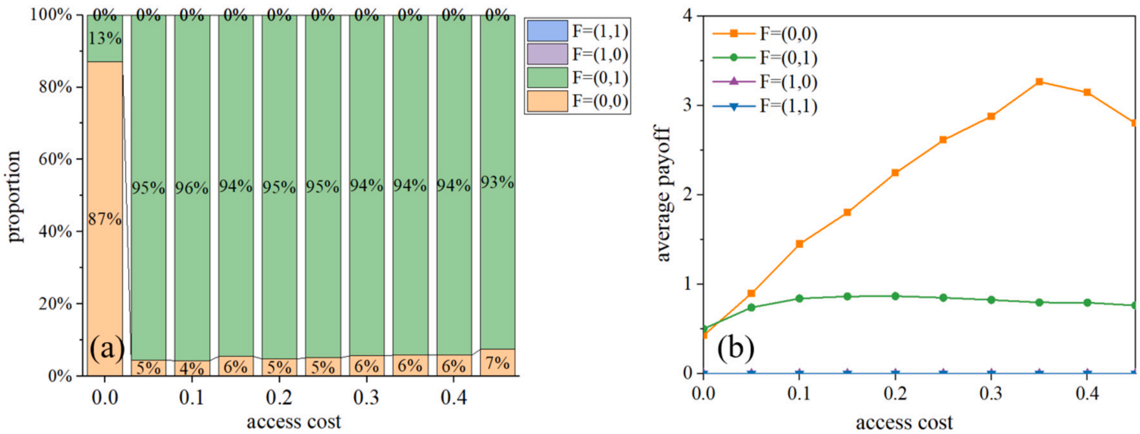

Fig. 5. Percentage of players and average payoffs of four payoff information use strategy populations on the Lattice network of degree 4 with payoff information use strategies when $k = 4$.

3.2. The impact of payoff information use strategy on social welfare

In this subsection, we study the phenomenon (2) of why the payoff information use strategy matters when $k > 1$, from the evolution of four populations with different payoff information use strategies: $F = (0,0)$, $F = (0,1)$, $F = (1,0)$ and $F = (1,1)$.

To study this phenomenon, as shown in Fig. 5 (a, b), we take $k = 4$ as an example and analyze the distribution of players and the average payoff of four populations on the Lattice network with payoff information use strategies. As shown in Fig. 5 (a), the payoff information use strategies of players eventually converge on two types: $F = (0,0)$ and $F = (0,1)$. Notably, the four populations converge on $F = (0,0)$ with 87% when $r = 0$ and converge on $F = (0,1)$ with more than 90% when $r \in (0,c)$. Interestingly, when we combine the average payoffs of the four strategies, we observe that the population with fewer players achieves a higher average payoff, as observed in Fig. 5 (b).

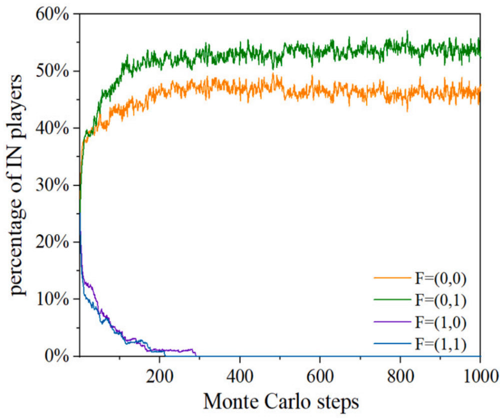

Fig. 6. The evolution process of the percentage of IN players of two populations among MC steps on the Lattice network of degree 4 with payoff information use strategies when $k = 4$ and $r = 0.25$.

To further investigate the interesting phenomenon presented above, we analyzed the evolutionary process of the proportion of IN players in four populations. As shown in Fig. 6, these populations were randomly distributed on the network at the onset of the evolutionary process. As the evolution progressed, players from populations $F = (1, 1)$ and $F(1, 0)$ were eliminated when Monte Carlo (MC) steps approached 200 and 300, respectively. The population $F = (0, 1)$ consistently maintained the highest number of IN players throughout the evolution. Interestingly, players from the population $F = (0, 0)$ were found to be more likely to become hubs with high payoffs and less likely to change strategies. On the other hand, players from the population $F = (0, 1)$ targeted these hubs as their access points despite being SA players, increasing the payoffs of hub players they accessed. Players who chose these hubs as their learning targets had a high probability of imitating their strategies and becoming new IN players. However, the payoffs of these newly-formed IN players were low, as only a few of their k-hop neighbors preferred to access them rather than the hubs. Thus, while the number of IN players with $F = (0, 1)$ was higher than those with $F = (0, 0)$, the average payoff was lower, further verifying the phenomenon observed in Fig. 5: because of the concentration of high-payoff IN players to $F = (0, 0)$ and the convergence of SA players on $F = (0, 1)$, namely specialization, four populations survival of the fittest.

In summary, compared to the network without payoff information use strategy, i.e., all players are $F = (0, 0)$ or $F = (1, 1)$, through the elimination of payoff information use strategies and the specialization of SA and high-payoff IN players, social welfare improves because lower-payoff players will learn and imitate higher-payoff players' strategy. Therefore, learning the payoff information use strategy can significantly improve the average social welfare, as presented in the phenomenon (2).

3.3. The impact of access costs on social welfare

In this subsection, we will analyze the phenomenon (3) from two aspects: (1) the intriguing observation that the average social welfare is greater than zero when $r = 0$, and (2) the trend of the average social welfare with access costs.

First, we concentrate on the interesting phenomenon that the average social welfare is greater than 0 when the $r = 0$. Generally, when $r = 0$, all players are strongly incentivized to free-ride since IN players cannot benefit from their k-hop neighbors. Therefore, the payoff of IN players must be lower than that of SA players. As a result, IN players are expected to eliminate as the evolution, and all players choose to free-ride, i.e., $\overline{U} = 0$. However, as Fig. 7 shows the evolution of the network with $k = 2$, we observe the proportion of IN players decreasing to 20% rather than 0%. The network snapshots at Monte Carlo steps 1, 10, 100, and 1000 reveal that FA players quickly spread outward as the evolution forms a large area. As IN players steadily grow, they begin to cut into the SA players' territory, causing them to disperse at the boundary between IN and FA players' territories. The existence of FA players slows down the elimination of IN players. After the evolution, both IN and SA players are present in the network, leading to an average social welfare greater than zero.

Next, we analyze the trend of the average social welfare with access costs. The change in the average payoff, which initially increases and subsequently decreases in response to increased access costs, is mainly affected by the emergence of hub IN players with high incomes. When access costs are low, players prefer to access IN players. As the evolution proceeds, players will concentrate on accessing some IN players, who generate higher payoffs than SA players. These IN players remain stable and attract other nodes, becoming hubs that enhance the average social welfare. However, when access costs continue to increase, IN players become more influenced by free-riding players. The potential stability of a hub decreases due to significant differences in income resulting from the changing accessing targets of a few SA players. Additionally, as access fees increase, the expected income from accessing other players declines, resulting in a reduced willingness to free-ride and an increased willingness to invest. The average social welfare, jointly affected by the proportion of IN and SA players, subsequently decreases.

To better show the change of this investment node, we assign the investment age attribute to all players and count it in each Monte Carlo step. When the player maintains investment, the age increases by 1, and when the player changes to free-riding, her age will reset to 0. As shown in Fig. 8, taking $k = 3$ as an example, we count the maximum, minimum, average, and distribution of investment age under different access costs. We observe that investment nodes have a minimum age of 0 and a maximum of 1000.

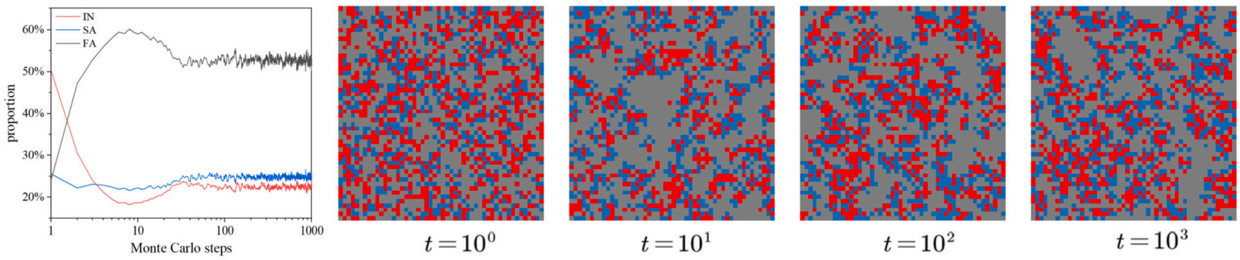

Fig. 7. The evolution process of the percentage of FA, SA, and IN players on the Lattice network of degree 4. And Snapshot of the evolution at Monte Carlo steps of 1, 10, 100, and 1000 when $k = 2$ and $r = 0$, where gray, blue, and red pixels indicate FA, SA, and IN players, respectively.

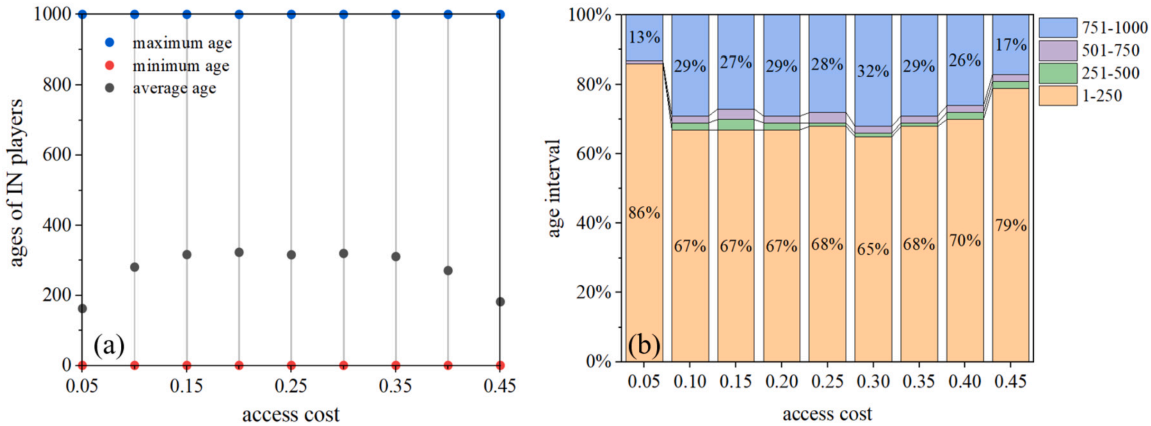

Fig. 8. Age statistics of investment nodes at k = 3 on the Lattice network of degree 4, (a) maximum, minimum, and average age, (b) proportion of IN players in different age intervals to all IN players.

The average age of investment nodes increases and decreases as access costs increase. Most IN players fall within the $[1, 250]$ age group and $[751, 1000]$ age group, with the $[1, 250]$ group having the highest number of IN players. This phenomenon satisfies that a few initial investors can become hubs, attracting other players to learn and access. As access fees continue to rise, the phenomenon that more IN players with an investment age higher than 500 satisfies the increasing possibility of forming hubs. However, with the further increase of access costs, just as with the increasing proportion of IN players in the $[251, 500]$ age interval, the perception of unstable hubs increases because the access of a few players can bring a high income.

In summary, with the growth of access cost, social welfare will increase first and then decrease because of the form of hub IN players. And especially when the access cost is about half the investment cost, social welfare is highest. Which explains the phenomenon (3) we observed.

4. Conclusion

In this paper, we analyzed the impact of different k-hops, payoff information use strategy, and access costs on social welfare in the k-hop evolutionary network public goods game model. Different types of players may go through different updating mechanisms, which introduces heterogeneity into our model and helps the network reciprocity that maintains social welfare behavior. Over time, the players' payoff information use strategy changes due to the impact of their historical payoffs. And compared to the basic evolutionary network public goods game model, which is equivalent to our model with $k = 1$, $r = 0$, and $F = (0, 0)$, our generalized model and two-step updating mechanism can significantly improve social welfare. Specifically, social welfare increases with diminishing marginal payoff as k increases because of the increasing range of common accessible targets. And compared to $k = 1$, introducing a payoff information use strategy can substantially improve social welfare when $k > 1$ by eliminating payoff information use strategies and the specialization of SA and high-payoff IN players. Besides, the access costs can also affect social welfare, and the improvement achieves the highest when the access cost is about half of the investment cost due to the form of hub IN players.

In summary, our study highlights the importance of the payoff information use strategy in enhancing social welfare in a Lattice network. Different from analyzing the performance of Nash equilibrium from the perspective of social welfare [8–14], and investigating the effect of incentive mechanisms from the perspective of cooperative emergence [36–38], we study the impact of strategic interaction of players from the perspective of social welfare and further explore the inner motivation of the emergence of social welfare from a micro perspective, which can effectively help researchers further analyze the formation of Nash equilibrium and the design of incentive-compatible mechanisms. We contribute to understanding the impact of k-hop, payoff information use strategy, and access cost on social welfare, providing insights for designing effective strategies in Best-shot public goods games. We hope this paper will contribute to further research of the evolutionary Best-shot public goods game and clearly explain the impact of payoff information use strategy on social welfare. Therefore, we plan to carry out more characteristics of heterogeneity, mechanisms, and network topology in the future. The details are as follows: (1) Study the heterogeneity of the investment [39] and access cost of players in our model, such as the banded charge of access cost based on k, i.e., the access cost grows as k increases; (2) Explore the incentive mechanisms, such as reward [36], punishment [37], and reputation [38] in our k-EBNPG model; (3) Analyze the effect of different network topologies on social welfare, such as the small world network [40] and the scale-free network [41].

Data availability

No data was used for the research described in the article.

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